DOI 10.14622/Advances_48_2022_18

Revising our thinking on tone value increase

John Seymour

John the Math Guy, LLC, and Clemson University
E-mail: john@johnthemathguy.com

Short abstract

The equation for tone value increase (TVI) is based on two things: 1) a simplistic mathematical model relating reflectance of a halftone and the area coverage, the Murray-Davies (MD) equation, and 2) the assumption that "dots get bigger" is an adequate explanation for the decrease in reflectance between plate and substrate. The TVI has proven a useful metric for process colors, but it has numerous drawbacks. It fails miserably for some spot colors. The calculated TVI depends on the wavelength. There is not a one-to-one correspondence between CIELAB values and TVI. The MD equation produces a poor estimate the spectrum of a halftone. These deficits all stem from the fact that MD is a simplistic mathematical model. Improvements to the MD mathematical model have been developed which add the effects of light scattering into the paper, and which account for the thinning of dots as they spread in the print process. A complete mathematical model would take both of these effects into account but since the two effects are similar numerically, either equation could be used as a stand-in for the combined effect of the two. This paper posits that the dot squish mathematical model could be used to generate a process control parameter that behaves similar to TVI. This parameter would work equally well with process and spot colors. Since it is based on a reasonably accurate mathematical model, it could be used to predict the spectra of halftones and hence there would be a one-to-one correspondence between this parameter and CIELAB values.

Keywords: tone value increase, TVI, Murray-Davies, Noffke-Seymour, process control metrics

1. History

1.1 The Murray-Davies equation

The computation of apparent tone value is based on an equation originally published by Alexander Murray in 1936 (Murray, 1936; Gamm, 2020). In this paper, he attributes the equation to Edward Davies – hence the equation has been named the Murray-Davies (MD) equation.

$$D = \log\left(\frac{1}{1 - a(1 - r)}\right) \qquad [1]$$

where D is the density of the halftone, a is the relative amount of paper covered with ink, and r is the reflectance of the solid.

A practical issue with this equation is that the reflectance of the paper is ignored. This was corrected in a 1941 paper by Yule (Yule, 1941; Dorst, 1943).

A didactic issue with Equation [1] is that it is obtuse. There is an underlying mathematical model, but that model is obscured by the math. A simpler form, first stated by Dorst, expresses the absorbance value of the halftone as a function of the dot area, and absorbance values of the paper and solid. Equation [2] shows Dorst's version, but in terms of reflectance rather than absorbance.

$$R_{\mathrm{t}} = (1 - A)R_{\mathrm{p}} + AR_{\mathrm{s}} \quad [2]$$

where R_p is the reflectance of the substrate (the paper), R_s is the reflectance of the solid, A is the area of the halftone, and R_t is the predicted reflectance of the halftone (the tint).

In his equation, it is seen that the reflectance of the halftone is a weighted average of the solid and the paper, with the weighting being the area of the dots and the uncovered paper, respectively. The underlying assumption is that a photon will either hit the ink (and reflect as if it were a solid) or hit paper (and reflect accordingly).

As an aside, the so-called Murray-Davies equation is an example of Stigler's law, which states that no scientific finding is named after the original inventors. Instead of Murray-Davies, the equation should be called Davies-Dorst.

1.2 Murray-Davies as a metric

The Murray paper is widely recognized as the source for the MD equation, but there was another contribution of this paper that is less well known. A large portion of this paper is devoted to understanding "dot spreading". Murray recognized that dots grew between the plate and the print, and that this growth led to changes in the density of the print. He proposed that, rather than use the actual size of the dot for process control, density would be a suitable proxy.

"For control of daily production, micrometry [direct measurement of dot size] is too laborious. As the object of the measurements is to secure data for the calculation of density, it seems more efficient to measure densities directly."

Murray planted the idea for a metric for dot gain, but did not provide a means to determine dot area from density measurements. This can readily be derived from the prediction provided by Equation [2]. Inverting this equation gives us a process control metric.

$$A_{\mathrm{out}} = \frac{R_{\mathrm{t}} - R_{\mathrm{p}}}{R_{\mathrm{s}} - R_{\mathrm{p}}} \quad [3]$$

If the reflectance values are all known from measurements of a press sheet, then A_{out} represents the *apparent dot area* of the halftone on the substrate. The difference between the dot area on the plate, A_p, and the apparent dot area from Equation [3] became known as the *dot gain*, A_Δ.

$$A_{\Delta} = A_{\mathrm{out}} - A_{\mathrm{in}} \quad [4]$$

More recently, the terms were renamed *apparent tone value* and *tone value increase* (TVI) to incorporate technologies where the dots were less distinct or absent. Thus, we have an indirect means to estimate the growth of halftone dots.

1.3 Deficiencies of Murray-Davies

The simplifying assumptions in the MD model have had some repercussions. Three of the repercussions are described here.

1.3.1 Wavelength dependence
According to the MD assumptions, it should be possible to compute the TVI at any wavelength where there is at least some absorption. The size of a dot should not depend on wavelength. The plot (Figure 1) shows

the TVI of a cyan halftone, computed at each individual wavelength. In this case, the TVI ranges from about 3 % to 18 %.

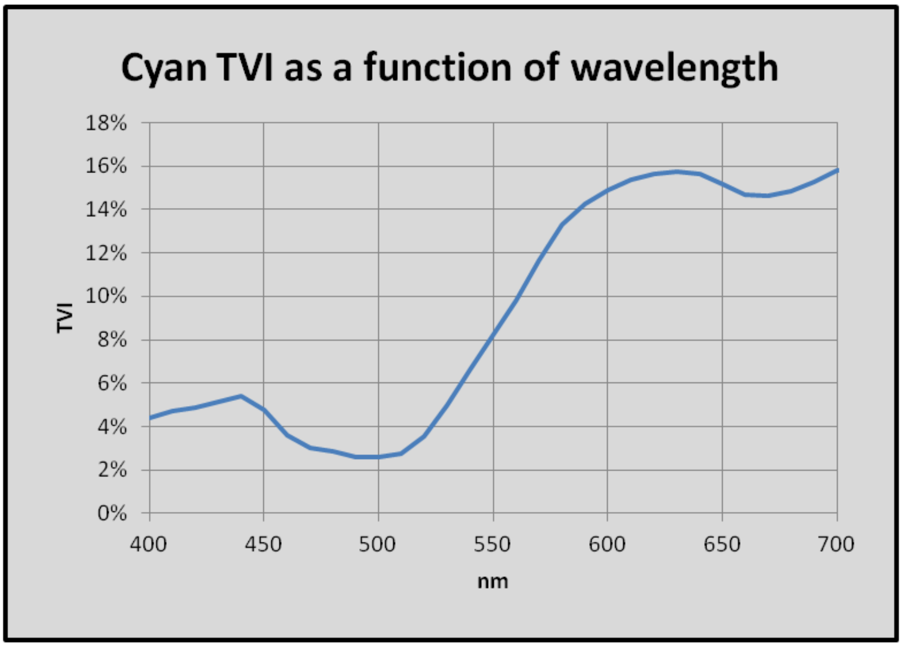

Figure 1: The TVI computed at each wavelength (Seymour, 2013c)

If the TVI were the same or nearly the same at all wavelengths, then it would not matter whether TVI were calculated with a Status T or E filter, or from *X*, *Y*, or *Z* values, or from narrowband reflectance. The numbers would be nearly the same. The choice of wavelength range would be a largely academic matter, rather than a potential source for miscommunication when TVI is computed from different parts of the spectrum.

From a theoretical standpoint, this serves as proof that MD does a poor job of simulating the reflectance of halftones. The size of a dot should not depend on what part of the spectrum you look at. Further, it is incorrect to refer to any of these measures of TVI as being a more accurate estimate of the "true TVI", since it is the model itself that is inaccurate.

From a practical standpoint the graph demonstrates that it is important to be consistent in the choice of wavelength range. Any of the wavelength ranges have been demonstrated to work in practice. For CMYK inks, TVI Status T/E broadband filters, Status I narrowband filters, and *XYZ* filters have all been demonstrated to work well for creation of plate curves and for process control.

1.3.2 Inability to predict spectra and color

If halftones followed the assumptions embedded in the MD equation, then it would be possible to reliably predict the spectrum of a given halftone from the spectra of the paper and the solid, along with the TVI. Figure 2 shows the problem with this.

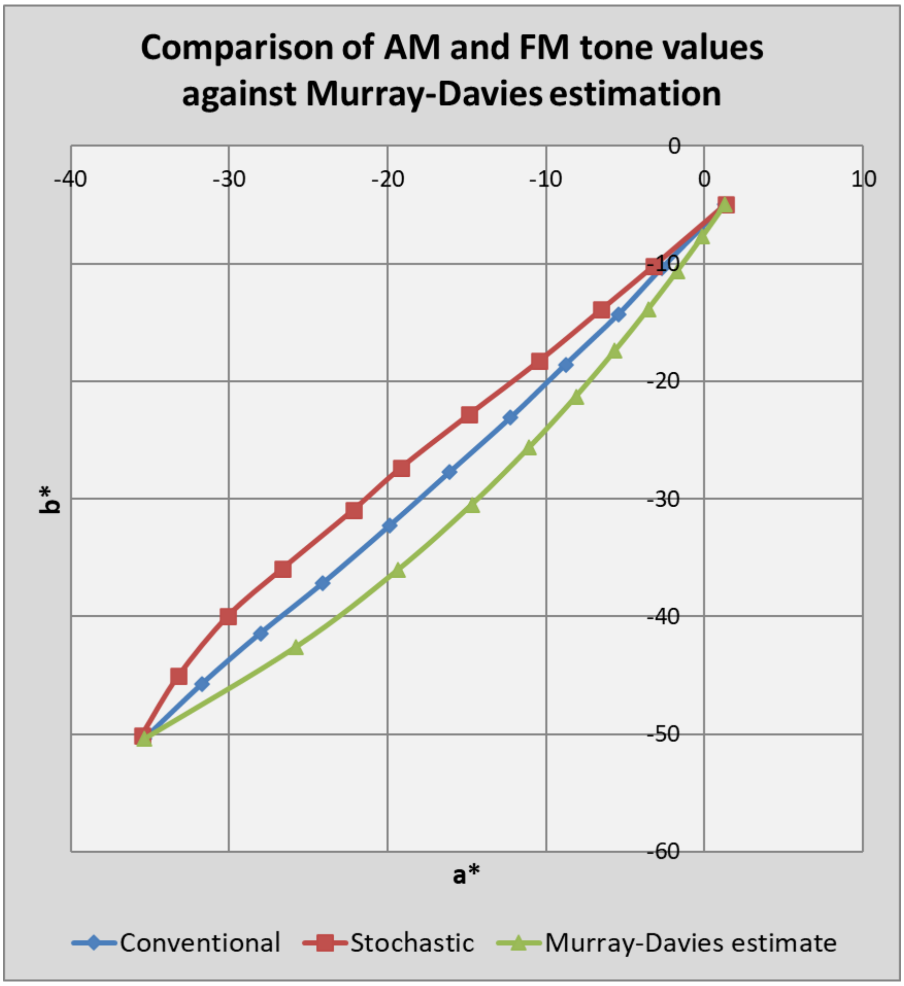

Figure 2: Murray-Davies predictions of halftones versus reality (Noffke and Seymour, 2012)

The plot shows the *a***b** values of three cyan tone ramps. The green curve traces the halftone trajectory of a hypothetical ink/press that follows the MD equation. The blue and red curves are actual data from cyan halftone ramps using conventional screening and stochastic screening on the same sheet. If we could rely on the MD equation to predict spectra, then we could rest assured that specifying a TVI would provide a reasonable color match.

1.3.3 Application to spot colors

In 2013, there was a growing awareness that TVI did not work well for spot colors. For certain inks, the attempt to linearize tone ramps using TVI failed abysmally. The lower bar graph in Figure 3 illustrates a tone ramp that is linear according to the MD equation. "That first step is a doozy!" The awareness of this problem led to the formation of the Spot Color Halftone Metric Optimization committee to find a better metric.

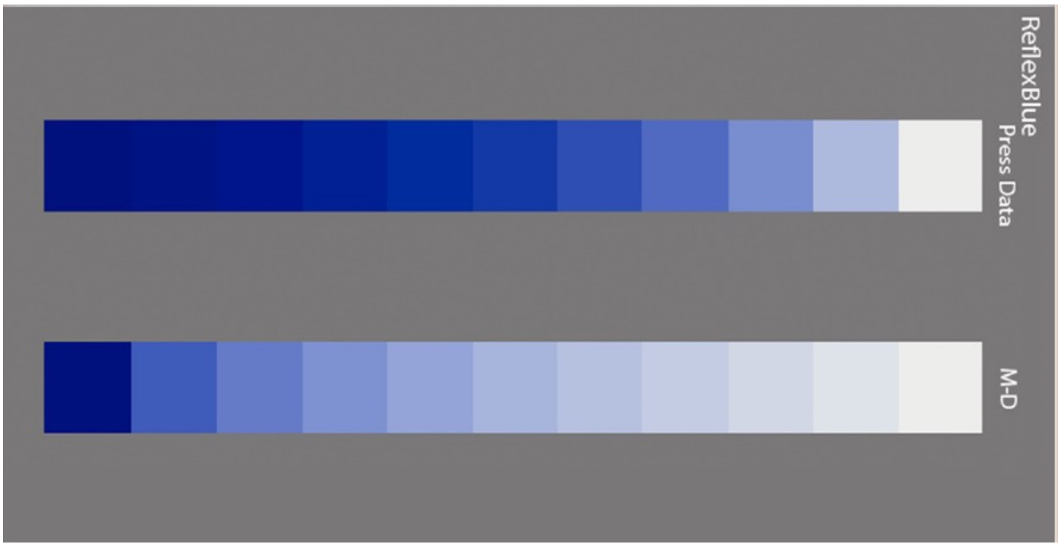

Figure 3: Reflex blue tone ramps, native (top) and after "linearizing with TVI" (below) (Smiley, 2015)

Many different equations were proposed and tested against real data. One of the equations was selected and has been formalized in ISO 20654 (International Organization for Standardization, 2017) under the name Spot Color Tone Value (SCTV). This new metric satisfied the immediate need for a way to construct linear tone ramps for spot colors. It was also found to be an acceptable replacement for TVI of process colors (Strickler, 2018).

SCTV does not, however, have an underlying mathematical model. As such, it cannot be used to estimate spectra of halftones (Seymour, 2013a; 2013b; 2013c; 2013d; 2014). Further, SCTV is only remotely related to colorimetry, so it cannot be counted on to provide perceptual linearity. Aside from the work done by the committee, there is no research that says that equal steps in SCTV are equal steps in perception.

It is the contention of this paper that the failure of MD TVI on spot colors is a direct consequence of the underlying simplistic mathematical model.

1.4 Improvements to the theory

Yule and Nielsen (1951) published a paper that showed that there were significant systematic differences between the actual dot area on a printed sheet and the apparent dot area as computed with the MD equation. For some reason, the dots on a printed sheet absorbed more light than would be expected from their size.

To explain the difference between theory and reality, Yule and Nielsen eliminated the second assumption in the MD equation, that the reflectance of the substrate in between the dots was the same as the bare substrate. They posited that some light entered through a halftone dot, scattered in the paper, and then exited outside of the halftone dot. In this way, the substrate between the dots was tinted toward the color of the solid. There would thus be an apparent increase in the area of the halftone dots. They called this *optical dot gain* (as opposed to *physical* dot gain), and provided an equation to predict this. The Yule-Nielsen (YN) model included a parameter n which related to the amount of light scattered in the substrate.

$$R_\mathrm{t}^{1/n} = (1 - A)R_\mathrm{p}^{1/n} + AR_\mathrm{s}^{1/n} \quad [5]$$

While this paper received much academic interest, it had little practical impact. The Yule-Nielsen n factor could have been used as a process control parameter in place of the dot gain parameter derived from the Murray-Davies equation. Two issues kept this from happening. The first issue was that their equation is unfortunately not invertible, so determining n for a given halftone would have required computers and iterative methods.

The second issue was that the paper ingrained the theoretical concept that “dot gain = physical dot gain + optical dot gain”. Ostensibly, physical and optical dot gain needed to be determined separately. It is likely that any idea of using the n factor to account for both optical and physical dot gain was squelched by the theoretical concept. And since physical dot gain was at best cumbersome to measure, the use of the YN equation was deemed impractical for common use.

So, industry continued to use the MD equation to determine the process control parameter that they called dot gain, largely oblivious to the fact that the control parameter might more accurately be called “increase in physical dot area plus the error caused by the use of the MD equation which neglects important physical phenomena in the mathematical model”.

The combination of MD and YN mathematical models is ineffective at predicting reflectance of midtone dots on a gravure press with very low viscosity inks. A midtone thus printed may spread to cover the entire area. MD would predict that the midtone has the same reflectance as the solid. Since there is effectively no paper showing through, there is no YN correction.

Noffke and Seymour (2012) provided a third mathematical model. Their model eliminated the assumption that the halftone dots have the same reflectance as the solid. They theorized that halftone dots are initially the same thickness as the solids, but as they transfer from plate to substrate, they spread out. Since the ink volume per dot must remain the same, there is a corresponding decrease in thickness corresponding to the increase in area.

Equation [5] is a restatement of the Noffke-Seymour (NS) equation from the original paper. This restatement is algebraically equivalent, but restates it in a way that the parameter A_Δ can be seen as analogous to TVI from the MD equation.

$$R_\mathrm{t} = (1 - A_\mathrm{in} - A_\Delta)R_\mathrm{p} + (A_\mathrm{in} + A_\Delta)R_\mathrm{p}\left(\frac{R_\mathrm{s}}{R_\mathrm{p}}\right)^{(A_\mathrm{in}+A_\Delta)/A_\mathrm{in}} \qquad [6]$$

It was shown that this mathematical model could be used to predict a variety of types of print, including ink jet printing and soft dot gravure which do not necessarily follow the intended model.

To summarize the three models, MD assumes that the reflectance of the paper between the dots is the same as the paper away from the dots and that the reflectance of the halftone dot is the same as that of the solid. YN eliminates the first assumption, and NS eliminates the second.

It would therefore seem reasonable that the most accurate model would eliminate both simplifying assumptions. The new definition of dot gain is “dot gain = NS dot gain + optical dot gain”. However, one of the findings in the Noffke-Seymour paper was that the YN and NS models produced very similar results. Because of this, it would be very difficult to look at a set of spectra and differentiate between the two effects. On the other hand, it is not generally necessary to determine which of the two effects dominates. Either one can be used to account for the effects of both.

Unfortunately, like the YN equation, the NS equation cannot be algebraically solved for A_Δ. An iterative algorithm must be used to determine the dot squish TVI.

2. To be demonstrated

The thesis of this paper is to demonstrate the following points:

1. The NS mathematical model can be used as a metric very similar to TVI computed from MD, but which is based on a more accurate mathematical model of the reflectance of halftones.
2. The NS TVI metric is less dependent on choice of wavelengths than is the MD model.
3. The NS TVI metric can provide a reasonable estimate of the spectrum of a halftone of a given dot area from the spectrum of the substrate and solid and the NS TVI.
4. The NS TVI metric will accurately predict the hue shift between stochastic and conventional halftones.

References

Dorst, P.W., 1943. Halftone densities. *Journal of Optical Society of America*, 3(8), pp. 436–439. https://doi.org/10.1364/JOSA.33.000436.

Gamm, B., 2020. The Murray-Davies equation: an origin story. *CMYK History* [blog], 18 October. Available at: <https://cmykhistory.com/murray-davies-equation-origin-story/> [Accessed February 2022].

International Organization for Standardization, 2017. *ISO 20654:2017 Graphic technology – Measurement and calculation of spot colour tone value*. Geneva: ISO.

Murray, A., 1936. Monochrome reproduction in photoengraving. *Journal of the Franklin Institute*, 221(6), pp. 721–744. https://doi.org/10.1016/S0016-0032(36)90524-0.

Noffke, P. and Seymour, J., 2012. A universal model for halftone reflectance. In: *64th TAGA Annual Technical Conference*. Jacksonville, FL, USA, 18–21 March 2012. Sewickley, PA: TAGA.

Seymour, J., 2013a. *Measuring TVI of a spot color* (unpublished). [pdf] Available at: <http://johnthemathguy.com/files/pdf/MeasuringTVIOfASpotColor.pdf> [Accessed February 2022].

Seymour, J., 2013b. The color of a bunch of dots, part 1. *John the Math Guy* [blog], 19 June. Available at: <https://johnthemathguy.blogspot.com/2013/06/the-color-of-bunch-of-dots-part-1.html> [Accessed February 2022].

Seymour, J., 2013c. The color of a bunch of dots, part 3. *John the Math Guy* [blog], 3 July. Available at: <https://johnthemathguy.blogspot.com/2013/07/the-color-of-bunch-of-dots-part-3.html> [Accessed February 2022].

Seymour, J., 2013d. The color of a bunch of dots, part 4. *John the Math Guy* [blog], 31 July. Available at: <https://johnthemathguy.blogspot.com/2013/07/the-color-of-bunch-of-dots-part-4.html> [Accessed February 2022].

Seymour, J., 2014. The color of a bunch of dots, part 5. *John the Math Guy* [blog], 25 June. Available at: <https://johnthemathguy.blogspot.com/2014/06/the-color-of-bunch-of-dots-part-5.html> [Accessed February 2022].

Smiley, S., 2015. *Managing spot colour tints*. FTA Forum 2015.

Strickler, M., 2018. Measuring CMYK Halftones: the case for spot color tone value formula. In: *70th TAGA Annual Technical Conference*. Baltimore, MD, 18–21 March 2018. Sewickley, PA: TAGA.

Yule, J.A.C., 1941. *Journal of the Franklin Institute*, 231(1), pp. 23–38. https://doi.org/10.1016/S0016-0032(41)90356-3.

Yule, J.A.C. and Nielsen, W.J., 1951. The penetration of light into paper and its effect on halftone reproduction. In: *Proceedings of the 3rd annual technical meeting.* Columbus, OH, 7–9 May 1951. TAGA, pp. 65–76.