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Discrete model of the inking apparatus of offset printing machine

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Abstract

Nowadays offset printing method is actively used in modern printing houses for printing graphic arts products, and makes it possible to obtain large print runs of high-quality full-color images. Offset printing technology allows to automate a large number of processes and due to this get a low cost of production, and equipment utilization rate. The purpose of this research is to create a scientific basis for structural and mathematical modeling of the inking apparatus of an offset printing machine. To achieve this goal, a discrete model of the ink apparatus was built as a control object associated with the movement and ink flow of the printing process, and the research results were summarized in the form of a mathematical model. Also, a methodology for the distribution of ink layers in the inking apparatus was developed. The inking apparatus is considered a dynamic system formed by a set of rollers and cylinders. The discrete model is based on the actual movement pattern of ink layers, considered as a directed graph. The sections of the trajectory of the ink layer movement are considered as arcs of the graph, and the contact points of rollers are considered as nodes of the graph, which correspond to the thickness of the ink layer for certain nodes and discrete moments of time. The necessary mathematical basis for computer modeling of an inking apparatus of complex structure has been created. The process of dividing the ink layer was described using difference equations. For computer implementation, based on the Gauss method of numerical solution of the systems of linear algebraic equations, software has been developed using the Visual Basic program, which allows to determine the divisions and thickness of the ink layer on the surfaces of the elements of the inking system. The discrete model with the help of difference equations describes the discrete process of division of the ink layer taking into account the time of displacement of the layer on the surfaces of rollers and cylinders. The result of modeling is cyclic processes of ink layer division, generalized in the form of dynamic characteristics. The extension of the proposed methodology to the inking apparatus of complex structure makes it possible to apply this methodology to the study of the inking apparatus of different designs.

Keywords: mathematical model, ink layer, difference equation, dynamic characteristic

1. Introduction

Analysis of the most common printing methods shows that the offset planographic printing method has the highest rate of development compared to other printing methods. This is due to the technical and economic advantages of the method and its better pictorial (rendering) capabilities. According to findings by Heidi Toliver-Nigro (2006), offset printing is the most common commercial printing method. This type of printing can nowadays reproduce almost any original while achieving high-quality printing performance. The increasing role of control intensively develops the automation of the functions of the printing press in the printing process. Automation increases the identity of circulation prints, contributes to the reduction of paper waste, and increases the equipment utilization rate. The basis for solving the problem of complex automation is the study of dynamic properties of the printing process as an object of automatic control and the generalization of the results of the study in the form of a mathematical model.

The works of many scientists have revealed various approaches to the mathematical description of inking apparatus. The overwhelming number of studies of the printing process is devoted to the study of the mechanism of layer-by-layer ink partitioning between contact surfaces. A large number of ink transfer modeling equations have been developed (Dai, et al., 2008; Zhao, 2011; Wu, Wu and Wang, 2011; Panichkin and Varepo, 2014; Aliyev, 2019; Verkhola, Panovuk and Huk, 2019; Verkhola, et al., 2022). Similarly, in order to determine the conditions for obtaining high-quality prints, a large number of methods for mathematical description of the printing process have been developed. One of the directions in the development of these methods is the mathematical study of the movement of ink and dampening solution in inking and dampening units. Using the specified constants, as well as taking into account the existing structure and design parameters of these devices it is possible to build their formal mathematical description.

The inking apparatus of offset printing machines is the most important technological unit, whose dynamic and static properties significantly affect printing quality. The inking unit forms a layer of ink of the required thickness for its subsequent transfer to the printing plate. For this purpose, the ink is fed by a ductor cylinder (ink fountain roller) from the ink fountain, rolled out by rollers, and rolled by rollers on the printing plate. To improve ink mixing in the ink box, a passive activator located parallel to the axis of the ductor cylinder is proposed, which is presented in the research paper by Litunov, Timoschenko and Gusak (2014).

A mathematical model of ink flow in the area between the squeegee (ink blade) and the ductor cylinder is presented, based on the model of non-viscous fluid flow. This model allows a visual assessment of the ink flow. However, this paper does not provide any information about the ink flow behavior in the contact zones of rollers and cylinders and separation in the contact zones of the elements of the ink-printing system. In some works (Liu, Lu and Bai, 2012; Liu, Li and Lu, 2016), the ink transfer model was modified based on the Reynolds equation under the condition of taking into account the retention of ink in the roller gaps. As a result of modeling, the thicknesses of the ink layer in the contact zones of the rollers are established. However, there is no information about the separation of the ink layer in the contact zones of the printing apparatus (print unit) cylinders. Zhao (2011) investigated the inconsistency of different structures and parameters of the inking system of modern offset printing press. The surfaces of ink rollers and cylinders were discretized by computer simulation, based on which a mathematical model of constant time in the periodic and continuous ink supply system was created. However, when creating a mathematical model, the issues of separating the ink layer in the contact areas of the elements of the ink printing system were not considered. The study by Yan, Hui, and Ling (2009) considered the inking system in offset printing as a complex undirected graph and constructed a network diagram. The researchers claim that by creating a dynamic two-dimensional array to record changes in ink thickness on an ink roller, the process of ink transfer during printing can be reliably and intuitively reproduced. However, the use of an undirected graph to determine the distribution of ink on an ink roller does not allow determining the actual pattern of movement of ink layers during discrete modeling of the ink apparatus. In the paper by Panichkin and Varepo (2014), ink flow between rotating cylinders and subsequent ink film breakdown were simulated using finite difference methods. The possibility of creating local areas of ink separation from the paper surface when leaving the print interaction zone is shown. However, in this work, when numerically calculating the movement of free boundaries, there is no information about the discrete modeling of the inking apparatus. In many works, the computer modeling method was used to study the inking system in offset printing (Dai, et al., 2008; Wu, Wu and Wang, 2011; Varepo, et al., 2018; Verkhola, Panovuk and Huk, 2019). Programs have been developed to simulate the flow of ink between cylinders. However, in these works, there is no information about the dynamic characteristics of the discrete model of the inking apparatus. Scientific publications (Aliyev, 2017; Aliyev, Khalilov and Ismailova, 2022) theoretically investigate ink transfer to the printing plate, taking into account the roughness of the surface of the printing plate. However, these works do not consider the modeling of the inking apparatus. The article by Aliyev (2019) considered the issues of modeling the inking apparatus of an offset printing machine. The model is built and the methodology for calculating the distribution of ink layers in the ink apparatus is developed. The regularity of the distribution of ink is established by calculating maximum and minimum layer thicknesses. However, the paper does not consider the regularity of ink distribution using discrete modeling of the ink apparatus. In a study by Verkhola, et al. (2022), a mathematical model of an offset inking and printing system was developed that describes the operating modes of all its components. The necessity of determining reliable values of ink-splitting coefficients in the contact zones of rollers and cylinders is substantiated. It is established that computer technologies allow for determining the amount of ink accumulated in the inking and printing system during printing, as well as the thickness and volume of ink on the surface of the prints. However, when creating a mathematical model, the dynamic characteristics of the inking apparatus are not considered.

The analysis of publications shows that there is no information about the discrete modeling of the ink apparatus in the process of ink transportation from the ink feeder to the plate cylinder in these works. Also based on the analysis of published scientific works, we can make the following conclusion about the state of the problem of mathematical modeling of the printing process:

- Traditional methods of mathematical analysis and theoretical mechanics cannot cover the study of complex and diverse dynamic phenomena of motion and interaction of the main material flows in offset printing. These methods also do not provide a complete mathematical description of the printing process as an object of automation. Therefore, it is desirable to develop mathematical methods of research based on software and hardware means of inking apparatus operation and their future model-oriented design.

The purpose of this work is to create a scientific basis for structural and mathematical modeling of the inking apparatus of offset printing machine. To achieve this goal, the following tasks were set:

- Construction of a discrete model of the inking apparatus as a control object related to the movement and flow of the ink in the printing process;
- Development of a methodology for the distribution of ink layers in the inking apparatus.

2. Materials and methods

2.1 Construction of a discrete model of the inking apparatus of an offset printing machine

The discrete model of the inking apparatus is based on the motion scheme of ink layers, i.e., in fact, the inking apparatus scheme, is considered as a directed graph. In this case, sections of the motion path of the ink layer are considered as branches of the graph, and the contact points of inks on the rollers are considered as nodes of the graph that correspond to the variables under consideration, i.e. thickness of the ink layer $h_i(n)$ for certain nodes i and discrete time instants n.

For each branch of the graph, two characteristics are introduced: 1) gear ratio β_i or $(1 - \beta_i)$; 2) bias factor α_i or $(1 - \alpha_i)$. Gear ratio β_i characterizes the proportion of the flow of ink transmitted in the forward direction after dividing the flow in the contact node; similarly gear ratio $(1 - \beta_i)$ characterizes the remaining fraction of the flow transmitted in the reverse direction. Bias factor α_i characterizes part of the ink path along the periphery of the roller between the contact nodes when the flow moves in the forward direction; bias factor $(1 - \alpha_i)$ characterizes, accordingly, the remaining part of the path when the flow moves in the opposite direction. In the work of Aliyev (2019) it is shown that in stationary mode, the amount of ink g_i, passing through each contact node in one printing cycle is equal to the amount of ink g_0, which is supplied to the inking apparatus on average per printing cycle and the amount of ink transferred to the impression, i.e. $g_0 = \delta_0 \gamma_0 = \cdots = \beta_{(i-1)} h_{(i-1)} - (1 - \beta_i) h_i = \beta_i h_i - (1 - \beta_{(i+1)}) h_{(i+1)} = \cdots = \sigma h$, where δ_0 is the pulse amplitude, γ_0 is the duty cycle, σ is the fill factor, h is the thickness of the ink layer on the impression. Consider a roller containing two nodes (Figure 1a). The outer layers of ink present between nodes, respectively, denote films of thickness f_{i-1} and f_i. The equations of the thickness of the layers of ink in the nodes of power and flow for the n-th turn will be:

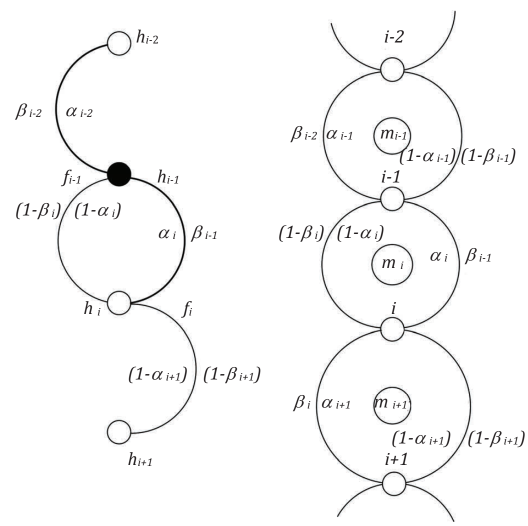
$$h_{i-1} = (1 - \beta_i)E^{-(1-\alpha_i)}h_i + f_{i-1} \quad [1]$$

$$h_i = \beta_{i-1}E^{-\alpha_i}h_{i-1} + f_i$$

where E is the offset operator (Korn and Korn, 1978), which is used to determine the transition of the ink layer from one node to another node. Excluding h_{i-1}, we get the equation of the roller, characterizing the thickness of the ink layer in the consumable node:

$$h_i = b_i E^{-1} h_i + \beta_{i-1}E^{-\alpha_i}f_{i-1} + f_i \quad [2]$$

where $b_i = \beta_i(1 - \beta_i)$.



a) b)

Figure 1: A discrete model of the ink apparatus; (a) – roller, (b) – baseline

Equation [2] is the basic equation of the ink roller, because it characterizes the dynamics of the ink layer, taking into account the division of the inner layer h_i at nodes and of contact receipts in these nodes from and to the outer layers f_{i-1} and f_i, respectively.

The case of considering the formation of the ongoing ink layer by a single node multiplicative operator E^{-1} and gear ratio combination b_i is insufficient if, in a real system, rollers have different rotation speeds. In this case, the calculated sampling period is chosen equal to the smallest of all the offset intervals: $\Delta t_p = (\Delta t_i)_{min}$.

All other offset intervals are expressed (rounded) by an integer number of calculated offset intervals. Consider the baseline of the ink apparatus, shown in Figure 1b. In the center of the closed trajectory of the ink flow along the surface of the roller (cylinder), an image of the auxiliary graph is given, characterizing the direction of the flow (clockwise or counterclockwise), the number is indicated m_i, showing how many calculated sampling periods correspond to the time of one turn of the roller, i.e. $m_i = \tau_i/\Delta t_p$, where $\tau_i = \pi d_i/V_f$ (d_i – roller diameter, V_f – ink flow rate). Given the number m_i of sampling periods, the given displacement coefficients are expressed for the forward and reverse directions, respectively: $r_i = \alpha_i m_i$, $(m_i - r_i) = (1 - \alpha_i)\, m_i$. The time (interval) of the displacement of the ink flow on the surface of the roller is expressed in terms of the displacement coefficient and the time of the full turn of the roller: $\Delta t_i = r_i \tau_i$. For some discrete moment of time $n = t/\Delta t_p$, thickness of the ink layer in the i-th contact node is formed by summing the two components (Figure 1b): 1) the thickness of the ink layer h_{i-1}, coming from $(i-1)$-th node in the i-th node in the direction of forward flow with a gear ratio β_i and negative bias (delay) $r_i = \alpha_i m_i$; 2) the ink layer thickness h_{i+1} coming from $(i+1)$-th in i-th node in the direction of return flow with a gear ratio $(1 - \beta_{i+1})$ and negative bias $(1 - \alpha_{i+1})\, m_{i+1}$.

Based on this, we write the equation of the contact node using the notation of the offset operator E:

$$h_i = \beta_{i-1} E^{-r_i} h_{i-1} - (1 - \beta_{i+1}) E^{-(m_{i+1} - r_{i+1})} h_{i+1} \qquad [3]$$

We represent Equation [3] in the form of a linear difference order equation r_i:

$$h_i E^{r_i} \equiv \beta_{i-1} h_{i-1} + (1 - \beta_{i+1}) E^{r_{i+1} + r_i - m_{i+1}} h_{i+1} \qquad [4]$$

It means that $r_i > (r_{i+1} - m_{i+1})$; otherwise, the equation is of order $(r_{i+1} - m_{i+1})$.

Solution of the difference equation of order r reduces in principle to solving a system r of first order difference equations. This way, we bring Equation [4] to a system of first order equations by introducing $h_i^{(r)} = E h_i^{(r-1)}$ as new variables so that $E h_i = h_i^{(1)}$:

$$E h_i^{(1)} = h_i^{(2)} \qquad [5]$$

$$\ldots$$

$$E h_i^{(r_i - 2)} = h_i^{(r_i - 1)}$$

$$E h_i^{(r_i - 1)} = \beta_{i-1} h_{i-1} + (1 - \beta_{i+1}) E^{r_{i+1} + r_i - m_{i+1}}$$

In a similar way, difference equations of the form presented in Equation [4] and equivalent systems of difference equations of the first order of the form in Equation [5] for nodes as parts of a complex ink apparatus, which can be given the serial numbers, can be written 1, 2,..., i,..., k, $k+1$, $k+2$, where k is the total number of rollers in the baseline, and the indices $k+1$, $k+2$ relate to plate and offset cylinders. General system of linear difference equations of the first order for all $k+2$ baseline nodes can be written in matrix form, which is parallel to the first-order linear difference equation:

$$EH = AH + F(n) \qquad [6]$$

where $H \equiv \left\{h_1, h_2, h_2^{(1)}, \ldots\right\}, F(n) \equiv \{h_0(n), \ldots\}$ are vectors (column matrices), and $A = [a_{ij}]$ square matrix of order S. The order of the matrix is determined by the total number $k+2$ nodes of the baseline, combined with the sum of the numbers of additional variables for individual nodes:

$$s = k + 2 + \sum_{i=1}^{z} (r_i - 1) \qquad [7]$$

where z is the number of individual nodes.

Matrix elements $[a_{ij}]$ are nonzero only for nodes directly connected by transmission lines (branches of the graph). For variables $h_i^{(r_i - 1)}$, connected only by the offset operator E, matrix elements are equal to unity. The solution of matrix Equation [6] with zero initial conditions is the vector

$$H = \sum_{n=0}^{m-1} A^{m-n-1} F(n) \qquad [8]$$

The powers of the matrix A needed to solve Equation [8] can be calculated by Cayley-Hamilton's theorem (Korn and Korn, 1978). For matrix order $s > 4$ finding a solution is associated with laborious calculations, and therefore it is advisable to use digital computers.

2.2 Discrete model of the inking apparatus of complex structure

Consider a discrete model as applied to the inking apparatus of complex structure (Figure 2). As the initial we take the thickness of the ink layer h_k = const on the surface of the duct cylinder. The transmission coefficients are assumed equal for all nodes, i.e. $\beta_i = \beta = 0.5$. In addition to the transfer coefficient, the ink flow transmitted to the plate cylinder is determined by the fill factor of the form by the printing elements. The interval of discreteness will take 1/4 of the time of turn of the form cylinder. The given displacement factors $r_i = \alpha_i m_i$ and coefficients m_i are expressed as rounded integers.

Applying the described methodology sequentially to all nodes of the inking apparatus as for the main (1, 2, 3, 4, 5, 6a, 7a, 8, 9), so for branches (2'; 6b; 7b; 4c, 5c, 6c, 7c), we compose the system of Equations [9].

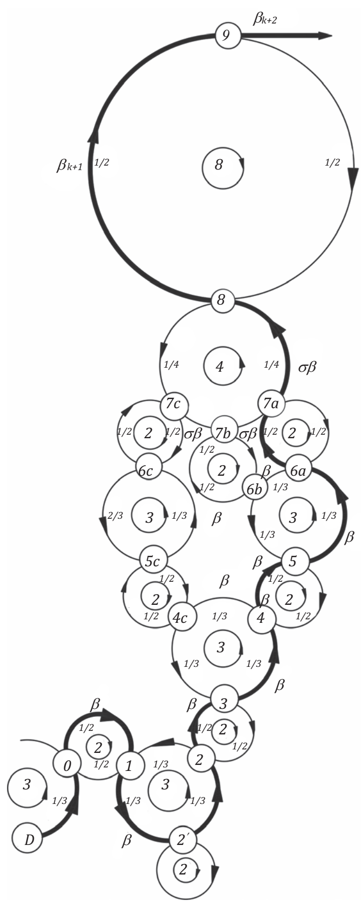


Figure 2: A discrete model of the inking apparatus of complex structure

$$\gamma h_0 = E^{-1}\gamma h_D + (1-\beta)E^{-1}\gamma h_1 \quad [9]$$
$$\gamma h_1 = \beta E^{-1}\gamma h_0 + (1-\beta)E^{-1}h_2$$
$$h'_2 = \beta E^{-1}\gamma h_1 + (1-\beta)E^{-2}h'_2$$
$$h_2 = \beta E^{-1}h_{2'} + (1-\beta)E^{-1}h_3$$
$$h_3 = \beta E^{-1}h_2 + (1-\beta)E^{-1}h_{4c}$$
$$h_4 = \beta E^{-1}h_3 + (1-\beta)E^{-1}h_5$$
$$h_5 = \beta E^{-1}h_4 + (1-\beta)E^{-1}h_{6b}$$
$$h_{6a} = \beta E^{-1}h_5 + (1-\sigma\beta)E^{-1}h_{7a}$$
$$h_{7a} = \beta E^{-1}h_{6a} + \sigma\beta E^{-1}h_{7b}$$
$$h_{6b} = \beta E^{-1}h_{6a} + (1-\sigma\beta)E^{-1}h_{7b}$$
$$h_{7b} = \beta E^{-1}h_{6b} + \sigma\beta E^{-1}h_{7c}$$
$$h_{4c} = \beta E^{-1}h_4 + (1-\beta)E^{-1}h_{5c}$$
$$h_{5c} = \beta E^{-1}h_{4c} + (1-\beta)E^{-1}h_{6c}$$
$$h_{6c} = \beta E^{-1}h_{5c} + (1-\sigma\beta)E^{-1}h_{7c}$$
$$h_{7c} = \beta E^{-1}h_{6c} + (1-\beta)E^{-1}h_8$$
$$h_8 = \sigma\beta E^{-1}h_{7a} + (1-\beta_{k+2})E^{-1}h_9$$
$$h_9 = \beta E^{-1}h_8$$

$$E\gamma h_0 = \gamma h_D + (1-\beta)\gamma h_1 \quad [10]$$
$$E\gamma h_1 = \beta\gamma h_0 + (1-\beta)h_2$$
$$Eh'_2 = h_{2'}^{(1)}$$
$$Eh_2 = \beta h_{2'} + (1-\beta)h_3$$
$$Eh_3 = \beta h_2 + (1-\beta)h_{4c}$$
$$Eh_4 = \beta h_3 + (1-\beta)h_5$$
$$Eh_5 = \beta h_4 + (1-\beta)h_{6b}$$
$$Eh_{6a} = h_{6a}^{(1)}$$
$$Eh_{7a} = h_{7a}^{(1)}$$
$$Eh_{6b} = \beta h_{6a} + (1-\sigma\beta)h_{7b}$$
$$Eh_{7b} = h_{7b}^{(1)}$$
$$Eh_{4c} = \beta h_4 + (1-\beta)h_{5c}$$
$$Eh_{5c} = \beta h_{4c} + (1-\beta)h_{6c}$$
$$Eh_{6c} = \beta h_{5c} + (1-\sigma\beta)h_{7c}$$
$$Eh_{7c} = \beta h_{6c} + (1-\beta)h_8$$
$$Eh_8 = h_8^{(1)}$$
$$Eh_9 = h_9^{(1)}$$
$$Eh_{2'}^{(1)} = \beta E\gamma h_1 + (1-\beta)h'_2$$
$$Eh_{7b}^{(1)} = \beta Eh_{6b} + \sigma\beta h_{7c}$$
$$Eh_{6a}^{(1)} = \beta Eh_5 + (1-\sigma\beta)h_{7a}$$
$$Eh_{7a}^{(1)} = h_{7a}^{(2)}$$
$$Eh_{7a}^{(2)} = \beta Eh_{6a}^{(1)} + \sigma\beta h_{7b}$$
$$Eh_8^{(1)} = h_8^{(2)}$$
$$Eh_8^{(2)} = h_8^{(3)}$$
$$Eh_8^{(3)} = \sigma\beta Eh_{7a}^{(2)} + (1-\beta_{k+2})h_9$$
$$Eh_9^{(1)} = h_9^{(2)}$$
$$Eh_9^{(2)} = h_9^{(3)}$$
$$Eh_9^{(3)} = \beta h_8$$

In the system of Equations [9] γ is the duty cycle, which is equivalent to pulsed power supply with period τ_0.

By introducing additional variables, we then obtain the system of first-order Equations [10].

2.3 Digital modeling of the inking apparatus

The task of modeling is to obtain the thickness of the layers in the contact zones depending on the displacement interval. The description of the process of dividing the ink layer using difference equations creates the necessary mathematical basis for digital modeling of the inking apparatus. Digital modeling is understood as such an organization of the computational process in a computer, which, in a certain sense, is similar to the process of dividing the ink layer in the inking apparatus. For computer implementation, software has been developed using the Visual Basic program, which, using macros, integrating with office programs, easily provides reproduction of graphic images of research results. The software is developed on the basis of the Gauss method for the numerical solution of systems of linear algebraic equations.

3. Results and discussion

Dynamic characteristics of the inking apparatus obtained as a result of digital modeling, i.e. the separation of the ink layer in the contact zones depending on the offset interval is shown in Figures 3 and 4. Figure 3a shows the beginning of the process (up to 2 rotations of the form cylinder), Figure 3b the continuation of the process (from 2 to 10 rotations of the form cylinder), and Figure 3c is a graphical representation of the process as a whole (from 2 to 20 rotations of the form cylinder) with continuous power supply. Figure 4a shows the beginning of the process (up to 2 rotations of the form cylinder), and Figure 4b the continuation of the process (from 2 to 20 rotations of the form cylinder) with pulsed power supply.

In continuous feeding (Figure 3), when the plate cylinder rotates up to 0.25 rotations, the thickness of the ink layer h_0 on the surface of the ductor cylinder, which contains the zero node of the baseline of the inking apparatus (Figure 2), reaches from 0 to 8 µm. This value remains stable up to 0.5 rotations of the plate cylinder. In the subsequent rotation of the plate cylinder up to 20 rotations, h_0 reaches up to 13 µm and the formation of the thickness of the ink layer on the surface of the ductor cylinder is stabilized. At 0.25 rotations of rotation of the plate cylinder, the formation of an ink layer h_1 begins on the surface of the first shaft, containing nodes 0 and 1 of the baseline. The layer thickness increases from 0 to 4 µm, which corresponds to 0.5 h_0. This value remains stable from 0.5 to 0.75 rotations of the plate cylinder. In further rotation of the plate cylinder up to 20 rotations, h_1 reaches up to 10.5 µm and the formation of the thickness of the ink layer on the surface of the first roller stabilizes.

As can be seen from the graph (Figure 3), the process of the beginning of the formation and gradual increase in the thickness of the ink layer from h_2 to h_6 on the surfaces of subsequent rollers, containing nodes 2, 3, 4, 5, 6a, of the baseline (Figure 2), occurs sequentially every 0.25 rotations of the plate cylinder and ends at 1.75 rotations of the plate cylinder. When the plate cylinder rotates from two rotations, the formation and gradual increase in the thickness of the ink layer h_7 occurs on the surface of the seventh shaft, which contains nodes 6a and 7a of the baseline (Figure 2). When the rotation of the plate cylinder reaches 20 rotations, the thickness of the ink layer h_7 reaches from 0 to 2 µm. At the same time, the process of stabilizing the formation of the thickness of the ink layer on the surfaces of all rollers also occurs.

With pulsed power supply (Figure 4), at 0.25 rotations of the plate cylinder, the thickness of the ink layer h_0 on the surface of the ductor cylinder of the inking apparatus, which contains the zero node of the baseline (Figure 2), reaches up to 8 µm. This value decreases to 0.25 µm when the plate cylinder rotates up to 2 rotations. In subsequent rotation of the plate cylinder up to 10 rotations, h_0 decreases to 0.07 µm. At 0.25 rotations of the plate cylinder, the formation of an ink layer h_1 begins on the surface of the first shaft, containing nodes 0 and 1 of the baseline. In this case, the thickness of the ink layer h_1 increases from 0 to 4 µm, which corresponds to 0.5 h_0. This value decreases from 0.5 to 2 cylinder rotations to 0.38 µm and remains stable up to 2 cylinder rotations. With further rotation of the plate cylinder up to 10 rotations, h_1 decreases to 0.07 µm.

As can be seen from the graph (Figure 4), the process of the beginning of the formation and gradual increase in the thickness of the ink layers h_2 to h_6 on the surfaces of subsequent rollers occurs sequentially, starting at 0.75 rotations of the plate cylinder, every 0.25 rotations of the plate cylinder and ending at 1.75 rotations of the plate cylinder. When the plate cylinder rotates up to 2 rotations, there is a gradual decrease in the thickness of the ink layer from h_2 to h_6 on the surfaces of the rollers containing nodes 2, 3, 4, 5, 6a, of the baseline (Figure 2). Also, the formation and increase in the thickness of the ink layer h_7 occurs on the surface of the seventh shaft, which contains nodes 6a and 7a of the baseline (Figure 2). When the plate cylinder rotates up to 5 rotations, the thickness of the ink layer h_7 reaches from 0 to 0.06 µm and decreases to 0.05 µm from 5 to 10 rotations of the plate cylinder. Also, with 10 rotations of the plate cylinder, the process of stabilizing the formation of the thickness of the ink layers on the surfaces of all rollers occurs.

Dynamic characteristics are approximated with high accuracy by an exponential function, which can be written in a logarithmic form. Introducing the notation

$$\theta = \frac{t}{T_0}\ , \theta_0 = \frac{\tau}{T_0}\ , x_i = \frac{h_i}{h_i^0} \qquad [11]$$

we represent the approximating function in two forms

$$x_i(\theta) = 1 - e^{\theta_0 - \theta}\ ; \quad \ln(1 - x_i) = \theta_0 - \theta \qquad [12]$$

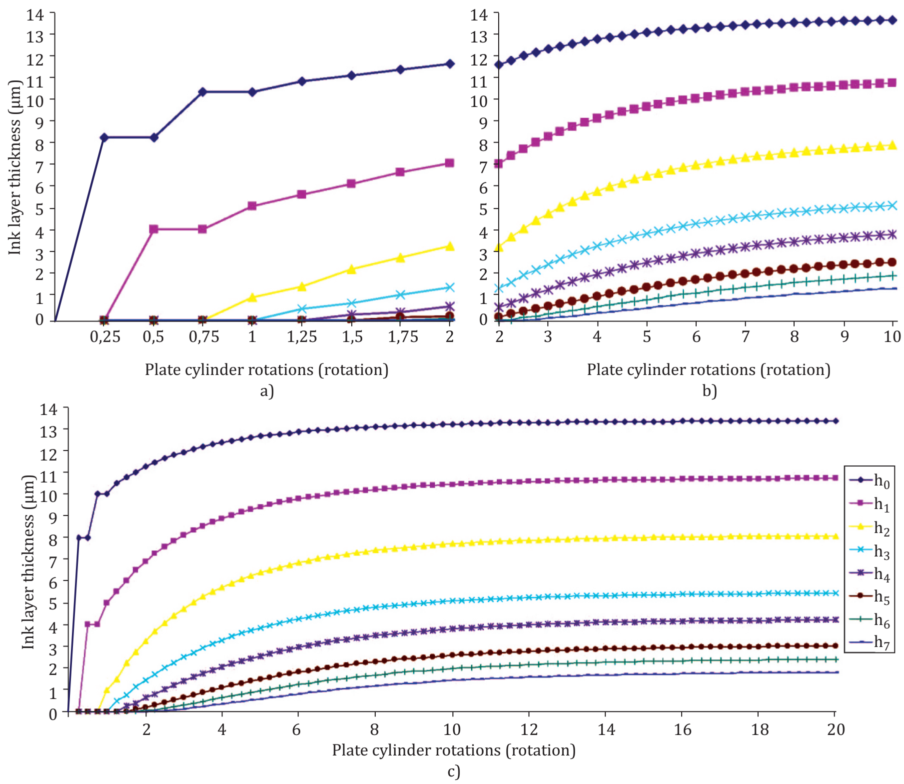


Figure 3: Dynamic characteristics of the inking apparatus at continuous power at the speed of the plate cylinder: (a) the beginning of the process; (b) continuation of the process; (c) a graphic representation of the process as a whole

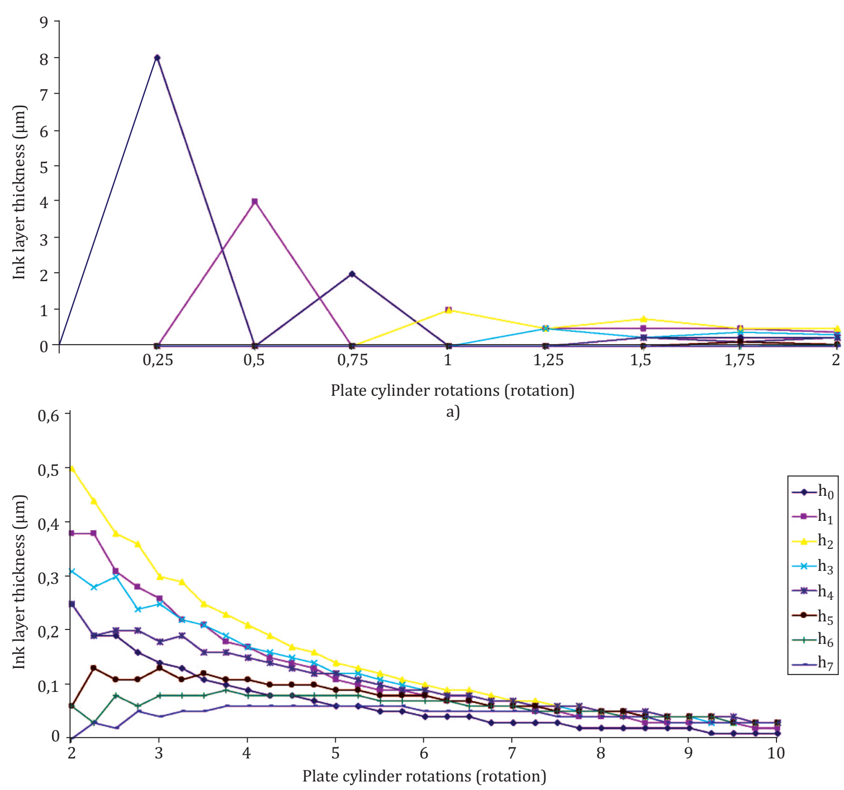


Figure 4: Dynamic characteristics of the inking apparatus at pulse power supply, with a rotation of the plate cylinder: (a) the beginning of the process; (b) continuation of the process

The exponential form directly follows from the simulation results. The logarithmic form gives a more accurate approximation, since it is expressed by a linear dependence.

An important property of the generalized output dynamic characteristic is that it does not depend on the number of rollers of the ink apparatus k, as well as on the transfer coefficient of ink to paper β_p. At the same time, the parameter of this characteristic is delay θ_0, coefficient dependent: $\theta_0 = 0.2(1 + 2\sigma)$. With increasing σ relative delay θ_0 increases and the characteristic shifts to the right along the abscissa.

Concretization of generalized output and internal dynamic characteristics is performed using dependencies $T_0 = T_0(k, \sigma, \beta_p)$, $h_0 = h_0(k, \sigma, \beta_p)$ and $\theta_0 = \theta_0(\sigma) = \theta_0(k, i)$, which are obtained on the basis of the results of digital modeling.

The dependence of the time constant T_0 on the number of rolls k at various values of σ (0; 0.5; 1) and β_p (0; 0.5; 1) is shown in Figure 5. The dependence approaches linear if we determine ln T_0.

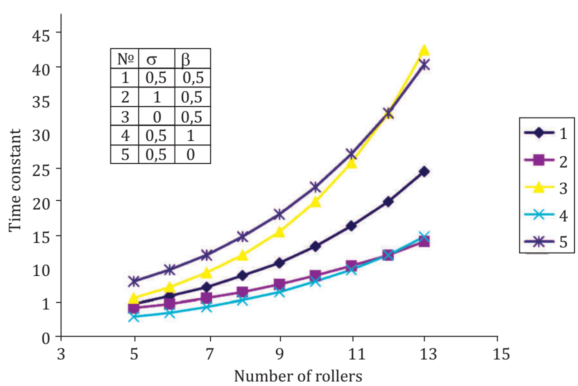


Figure 5: The dependence of the time constant T_0 on the number of rollers k in inking apparatus

Based on the data of digital modeling, an empirical formula can be proposed

$$\ln T_0 = 1.5 - \beta_p + 0.25(1 - 0.4\sigma)(k - 2) \qquad [13]$$

allowing to determine T_0 at given values k, σ, β_p.

The generalized internal dynamic response at $\sigma = 0$ reflects the dynamics of the inner layers of the ink apparatus and also depends on the parameter θ_0, which in this case can characterize either a delay or a lead. At a certain number of the inner layers i_0 characteristic goes through zero $\theta_0 = 0$. At $k \geq i > i_0$ there is a delay $\theta_0 > 0$; at $1 < i < i_0$ it is a lead $\theta_0 < 0$. For each number of rollers k in the ink apparatus there will be a value i_0, which increases with increasing k.

4. Conclusions

To investigate the process of ink transfer and distribution occurring in the ink-printing system of an offset machine in the printing process, a discrete model of the ink apparatus related to the movement and ink flow of the ink apparatus is constructed. A method for determining the distribution of ink layers in the ink apparatus, using computer technology, is developed. For the basis of the discrete model of the ink apparatus, the scheme of movement of ink layers, which corresponds to the actual scheme of the ink apparatus, is taken. This scheme was considered as a directed graph. Sections of the trajectory of the ink layer were considered as arcs of the graph. In this case, the contact points of the rollers were taken as nodes of the graph, which correspond to the thickness of the ink layer $h_i(n)$ for certain nodes i and discrete time points n. To characterize the fraction of the ink flow transmitted in the forward direction after dividing the flow in the contact node and also the remaining fraction of the flow transmitted in the opposite direction for each branch, the transfer coefficient β_i or $(1 - \beta_i)$, respectively, is introduced into the graph. To characterize the part of the ink path along the periphery of the roller between the contact nodes when the flow is in the forward direction and the remaining part of the path when the flow is in the reverse direction, a displacement factor α_i or $(1 - \alpha_i)$ is introduced for each branch of the graph, respectively. The discrete model by means of difference equations describes the discrete process of ink layer partitioning considering the layer displacement time along the rollers surface. The proposed methodology for the distribution of ink layers can be extended to ink apparatus of complex structure having branched ink streams. In this case, the thickness of the ink layer h_k = const on the surface of the ductor cylinder is taken as the initial one. Transmission coefficients are assumed equal for all nodes, $\beta_i = \beta = 0.5$. The discreteness interval is taken to be 1/4 of the rotation time of the plate cylinder. The given displacement coefficients $r_i = \alpha_i m_i$ and coefficients m_i are expressed as rounded integers. Dynamic characteristics of the inking apparatus, i.e. separation of the ink layer in the contact zones depending on the displacement interval is considered with continuous and pulsed power. It has been established that in continuous feeding, when the plate cylinder rotates from 0.25 to 2 rotations, every 0.25 rotations, there is a consistent formation and gradual increase in the thickness of the ink layer on the surfaces of all the shafts of the ink apparatus along the baseline. When the rotation of the plate cylinder reaches 20 rotations, the process of stabilizing the formation of the thickness of the ink layer on the surfaces of all rollers along the baseline occurs. With pulsed power, when the plate cylinder rotates from 0.25 to 1.75 rotations, every 0.25 rotations there is a sequential formation and gradual increase in the thickness of the ink layers on the surfaces of all baseline shafts, with the exception of the seventh shaft. When the rotation of the plate cylinder reaches two rotations, a gradual decrease in the thickness of the ink layer on the surfaces of these rollers begins. From this moment, the thickness of the ink layer forms and increases on the surface of the seventh shaft up to 5 rotations and gradually decreases, starting from 5 rotations of the plate cylinder. When the rotation of the plate cylinder reaches 10 rotations, the formation of the thickness of the ink layers on the surfaces of all rollers along the baseline is stabilized. The considered discrete model of the inking apparatus is a refinement and development of the previously considered single-capacity model, which increases the reliability of the final result. The proposed method makes it possible to determine the thickness of the ink layer on the print and the amount of ink required to print an edition, taking into account the amount of ink accumulating in the inking system during the printing process. To obtain reliable information about the size of the ink layer on a print during printing, it is necessary to develop an appropriate technique. To test this technique, experimental studies are required.

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